Statistics

Fall 2022

Lecture 14

Feb 19-8:47 AM

Multiplication Rule

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

A happens then B happens

Given

Dependendent Events.

A box has 4 Red & 6 Blue balls. Randomly draw 2 balls without replacement, find

$$P(2 \text{ Red balls}) = P(R \text{ and } R) = \frac{4}{10} \cdot \frac{3}{9} = \frac{12}{90} = \frac{4}{30} = \boxed{\frac{2}{15}}$$

$$P(2 \text{ Blue balls}) = P(B \text{ and } B) = \frac{\cancel{6}}{\cancel{10}} \cdot \frac{\cancel{5}}{\cancel{9}} = \boxed{\frac{1}{3}}$$

Nov 15-6:02 AM

A standard deck of playing cards has 52 cards, 26 are red, 12 are face, and 4 Aces. Let's draw 3 cards **without replacement.**

$$P(\text{All Red}) = \frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} = \boxed{\frac{2}{17}}$$

$$P(\text{All face cards}) = \frac{12}{52} \cdot \frac{11}{51} \cdot \frac{10}{50} = \boxed{\frac{11}{1105}}$$

$$P(\text{All Aces}) = \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = \boxed{\frac{1}{5525}}$$

Nov 15-6:08 AM

A piggy bank has 2 dimes and 6 nickels. Shake it to get 2 coins. (**No replacement**)

D → Dimes , N → Nickels

Sample Space

NN → 10¢

ND, DN → 15¢

DD → 20¢

$$P(\text{Total} = 10¢) = P(NN) = \frac{6}{8} \cdot \frac{5}{7} = \boxed{\frac{30}{56}}$$

$$P(\text{Total} = 15¢) = P(ND \text{ OR } DN) = 2\left(\frac{6}{8} \cdot \frac{2}{7}\right) = \boxed{\frac{24}{56}}$$

$$P(\text{Total} = 20¢) = P(DD) = \frac{2}{8} \cdot \frac{1}{7} = \boxed{\frac{2}{56}}$$

Verify that total Prob. = 1 ✓ $\frac{30}{56} + \frac{24}{56} + \frac{2}{56} = \frac{56}{56}$

Total	P(Total)
10¢	30/56
15¢	24/56
20¢	2/56

Total → L1, P(Total) → L2

Use 1-Var Stats with List: L1 , FreqList L2

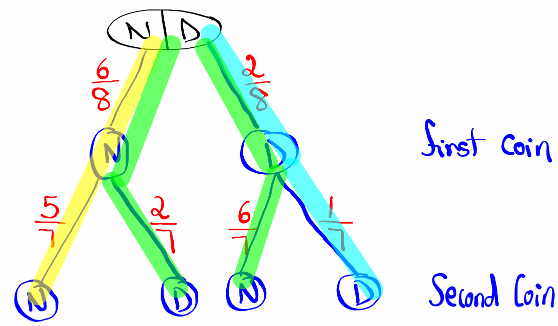
$\bar{x} = 12.5$

S_x = blank

Total Prob. = 1 → n = 1

Nov 15-6:17 AM

Draw Tree Diagram for last example:



$P(NN) = \frac{6}{8} \cdot \frac{5}{7} = \frac{30}{56} = \boxed{\frac{15}{28}}$ $\quad P(DD) = \frac{2}{8} \cdot \frac{1}{7} = \frac{2}{56} = \boxed{\frac{1}{28}}$

$P(\text{1 of each}) = P(ND \text{ or } DN) = 2 \cdot \frac{6}{8} \cdot \frac{2}{7} = \boxed{\frac{3}{7}}$

Verify total Prob.=1 ✓ $\quad \frac{15}{28} + \frac{1}{28} + \frac{3}{7} = 1$

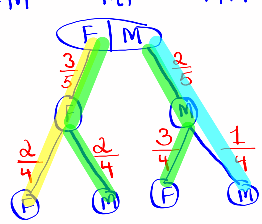
15 ÷ 28 + 1 ÷ 28 + 3 ÷ 7 Enter

Nov 15-6:30 AM

$\boxed{\text{3 Females, 2 Males}}$ Select 2 different people

Sample Space $\quad$ F → female, M → Male $\quad$ No replacement

FF $\quad$ FM $\quad$ MF $\quad$ MM



$P(\text{2 females}) = P(FF) = \frac{3}{5} \cdot \frac{2}{4} = \boxed{.3}$

$P(1F \,\&\, 1M) = P(FM \text{ or } MF) = 2 \cdot \frac{3}{5} \cdot \frac{2}{4} = \boxed{.6}$

$P(\text{No Females}) = P(MM) = \frac{2}{5} \cdot \frac{1}{4} = \boxed{.1}$

# Females	P(# Females)
2	.3
1	.6
0	.1

Females → L1
P(# Females) → L2
Use 1-Var Stats with L1 & L2 to find

$\bar{x} = 1.2$

$S_x =$ blank

Total Prob.=1 → n = 1

Nov 15-6:38 AM

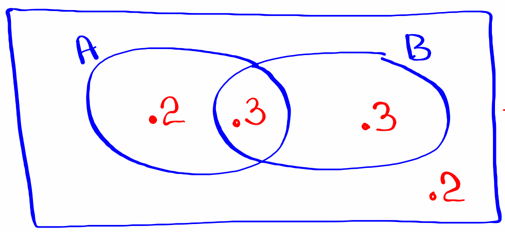
Class QZ 14:

$P(A) = .5$ $\quad$ $P(B) = .6$ $\quad$ A & B are independent events.

1) $P(A \text{ and } B) = P(A) \cdot P(B) = (.5)(.6) = \boxed{.3}$

2) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .5 + .6 - .3 = \boxed{.8}$



Total = 1

Nov 15-6:51 AM

Conditional Probability:

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

If we solve for $\boxed{P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)}}$

Suppose $P(A) = .4$, $P(B) = .5$, $P(A \text{ and } B) = .3$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{.3}{.4} = \boxed{.75}$$

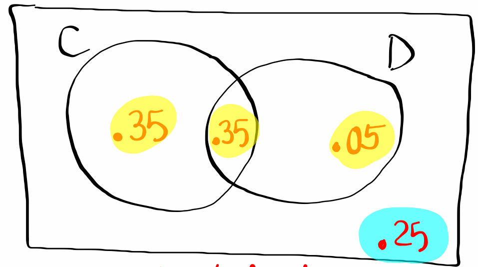
$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.3}{.5} = \boxed{.6}$$

Nov 15-7:12 AM

$P(\text{Coffee}) = .7$ ✓

$P(\text{Donut}) = .4$ ✓

$P(\text{coffee and Donut}) = .35$



Total = 1

$$P(\text{Donut} \mid \text{Coffee}) = \frac{P(\text{Coffee and Donut})}{P(\text{Coffee})} = \frac{.35}{.7} = \boxed{.5}$$

$$P(\text{Coffee} \mid \text{Donut}) = \frac{P(\text{coffee and donut})}{P(\text{Donut})} = \frac{.35}{.4} = \boxed{.875}$$

Nov 15-7:17 AM

Combination Formula

$${}_nC_r = \frac{n!}{r! \cdot (n-r)!}$$

$${}_6C_2 = \frac{6!}{2! \cdot (6-2)!} = \frac{6!}{2! \cdot 4!} = \frac{\overset{3}{\cancel{6}} \cdot 5 \cdot \cancel{4!}}{\cancel{2} \cdot 1 \cdot \cancel{4!}} = \boxed{15}$$

Using TI

6 [MATH] [→] PRB [↓] [nCr] 2 [Enter] 15

Find ${}_8C_3$

8 [MATH] [→] PRB [↓] [nCr] 3 [Enter] $\boxed{56}$

Find ${}_{300}C_6$

300 [MATH] [→] PRB [↓] [nCr] 6 [Enter]

$\approx \boxed{9.6 \times 10^{11}}$ E11

Scientific Notation

Nov 15-7:26 AM

What is $_nC_r$?

n different objects

Take r of these objects, No replacement

and order does not matter

$_nC_r \rightarrow$ # of ways this can be done.

A deck of Cards has 52 Cards, and 12 face Cards.

1) How many ways can we draw 3 Cards?

$_{52}C_3 = \boxed{22100}$ ✓

2) How many ways can we draw 3 face Cards? $_{12}C_3 = \boxed{220}$

3) $P(3 \text{ face Cards}) = \frac{_{12}C_3}{_{52}C_3} = \frac{220}{22100} = \boxed{\frac{11}{1105}}$

Nov 15-7:35 AM

A piggy bank has $\boxed{\text{5 dimes and 15 nickels}}$

Randomly get 2 Coins. No replacement

Order does not matter

1) # ways to get 2 coins

$_{20}C_2 = \boxed{190}$

2) # ways to get 2 dimes

$_5C_2 = \boxed{10}$

3) $P(2 \text{ dimes}) = \frac{_5C_2}{_{20}C_2} = \frac{10}{190} = \boxed{\frac{1}{19}}$

If we draw 3 coins

$P(\text{All nickels}) = \frac{_{15}C_3}{_{20}C_3} = \frac{455}{1140} = \boxed{\frac{91}{228}}$

Nov 15-7:41 AM

4 Females & 8 Males

Select 4 people, order does not matter.

1) how many ways can we Select 4 people?

$$ {}_{12}C_4 = \boxed{495} $$

2) how many ways can we Select 2 of each?

Females * # Males

$$ {}_4C_2 \cdot {}_8C_2 = \boxed{168} $$

3) $P(2F \text{ \& } 2M) = \dfrac{{}_4C_2 \cdot {}_8C_2}{{}_{12}C_4} = \dfrac{168}{495} = \boxed{\dfrac{56}{165}}$

Nov 15-7:48 AM

A standard deck of playing Cards has 52 Cards, 26 Red, 12 Face, and 4 Aces.

Randomly draw 5 Cards, No replacement

$P(3 \text{ Face \& } 2 \text{ Aces}) = \dfrac{{}_{12}C_3 \cdot {}_4C_2}{{}_{52}C_5} = \dfrac{132\cancel{0}}{259896\cancel{0}}$

$= 5.1 \times 10^{-4}$

$\dfrac{132}{259896} = \dfrac{66}{129948} = \dfrac{33}{64974}$

$= \dfrac{\cancel{3} \cdot 11}{\cancel{3} \cdot 21658}$

$= \boxed{\dfrac{11}{21658}}$

Nov 15-7:53 AM

Prob. with at least 1:

$$\boxed{P(\text{at least } 1) = 1 - P(\text{None})}$$

3 Females, 7 Males, Select 2 people
No replacement
Order does not matter

F → Females, M → Males

FF
FM
MF
MM

$P(\text{at least 1 Female})$

$= 1 - P(\text{No Female})$

$= 1 - P(MM)$

Total Prob. = 1

$= 1 - \frac{7}{10} \cdot \frac{6}{9} = \boxed{\frac{8}{15}}$

FF
FM
MF
MM

$P(\text{at least 1 Male})$

$= 1 - P(\text{No Male})$

$= 1 - P(FF) = 1 - \frac{3}{10} \cdot \frac{2}{9}$

$= \boxed{\frac{14}{15}}$

Nov 15-8:03 AM

5 nickels 15 pennies

Select 4 Coins, No replacement

NNNN

Some N & Some P

PPPP

$P(\text{at least 1 penny})$

$= 1 - P(\text{No pennies})$

$= 1 - P(\text{All nickels})$

$= 1 - \frac{5}{20} \cdot \frac{4}{19} \cdot \frac{3}{18} \cdot \frac{2}{17}$

$= \boxed{\frac{968}{969}}$

Nov 15-8:15 AM

Class QZ 15:

Given $P(E) = .2$

1) $P(\bar{E}) = 1 - P(E) = 1 - .2 = \boxed{.8}$

2) odds in favor of event E.

$P(E) : P(\bar{E}) \rightarrow .2 : .8 \rightarrow \boxed{1:4}$

3) odds against event E. $\longrightarrow \boxed{4:1}$

Nov 15-8:19 AM